Coupled-Mode Formulation of Multilayered and Multiconductor Transmission Lines

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***Abstract*— A novel coupled-mode formulation for multilayered and multiconductor transmission lines is developed. In this formulation, the solutions to the original multiconductor system are approximated by a linear combination of eigenmode solutions associated with the isolated single conductor line located in an appropriate reference dielectric medium, and the reciprocity theorem is used to derive the coupled-mode equations. The coupling coefficients are expressed in terms of the simple overlap integrals between the eigenmode fields and currents of the individual conductor lines. As a basic application, the dispersion characteristics of two identical coupled-microstrip lines are analyzed using the proposed coupled-mode theory. It is shown that the results are in very close agreement with those obtained by the direct Galerkin's moment method over a broad range of weak to strong coupling.**

I. Introduction

Multiconductor transmission lines arranged in a multilayered dielectric medium are widely used in the design of microwave and millimeter-wave integrated circuits. One of the most important subjects on such transmission systems is to evaluate efficiently as well as accurately the high-frequency electromagnetic coupling between nearby conductor lines. The coupling causes a crosstalk that seriously affects the circuit performance in high-speed operation. The transmission characteristics of coupled conductor lines can be rigorously analyzed using various numerical methods [1]. However, those direct solution methods become much more involved both analytically and numerically when the number of conductor lines increases and the conductors are situated in different layers of the multilayered dielectric medium. Therefore, there has been a strong need to implement approximate solution methods [2], [3] for multilayered and multiconductor transmission lines.

In this paper, we present a coupled-mode formulation for multilayered and multiconductor transmission lines based on the full-wave analysis. The formulation is an extension of the corresponding coupled-mode approach [4] for optical waveguides. The total fields supported by multiconductor lines are approximated by a linear combination of the modal fields associated with the isolated single conductor lines, and the coupled-differential equations governing the evolution of modal amplitudes in each line are derived by making use of the reciprocity relation. The coupling coefficients are given by the simple overlap integrals between the eigenmode fields and currents of individual conductor lines in isolation. The new formulation is applied to the analysis of two identical coupled microstrip lines. It is shown that the dispersion characteristics calculated by the present coupled-mode theory are in very close agreement with those obtained from the direct Galerkin's moment method in the spectral domain over a broad range of the separation distance between the two microstrips.

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II. Formulation

Let $\boldsymbol{E}_1$ and $\boldsymbol{H}_1$ be the electric and magnetic fields produced by a current source $\boldsymbol{J}_1$ located in one dielectric medium with permittivity $\varepsilon_1(y)$ and permeability μ_0, and $\boldsymbol{E}_2$ and $\boldsymbol{H}_2$ be the electric and magnetic fields produced by a current source $\boldsymbol{J}_2$ located in another dielectric medium with permittivity $\varepsilon_2(y)$ and permeability μ_0. These electromagnetic fields satisfy Maxwell's equations

$$\nabla \times \boldsymbol{E}_1 = -j\omega\mu_0\boldsymbol{H}_1 \tag{1}$$

$$\nabla \times \boldsymbol{H}_1 = j\omega\varepsilon_1(y)\boldsymbol{E}_1 + \boldsymbol{J}_1 \tag{2}$$

$$\nabla \times \boldsymbol{E}_2 = -j\omega\mu_0\boldsymbol{H}_2 \tag{3}$$

$$\nabla \times \boldsymbol{H}_2 = j\omega\varepsilon_2(y)\boldsymbol{E}_2 + \boldsymbol{J}_2 \tag{4}$$

and boundary conditions in the respective configurations. Using a similar procedure for the Lorentz reciprocity theorem in (1)-(4), the following equation is derived

$$\nabla \cdot (\boldsymbol{E}_1 \times \boldsymbol{H}_2 - \boldsymbol{E}_2 \times \boldsymbol{H}_1) = j\omega\Delta\varepsilon(y)\boldsymbol{E}_1 \cdot \boldsymbol{E}_2 + \boldsymbol{E}_2 \cdot \boldsymbol{J}_1 - \boldsymbol{E}_1 \cdot \boldsymbol{J}_2 \tag{5}$$

where $\Delta\varepsilon(y) = \varepsilon_1(y) - \varepsilon_2(y)$. When (5) is applied to a cylindrical geometry which is translationally invariant in the z direction, we obtain

$$\frac{\partial}{\partial z}\int_S (\boldsymbol{E}_1 \times \boldsymbol{H}_2 - \boldsymbol{E}_2 \times \boldsymbol{H}_1) \cdot \hat{\boldsymbol{z}}dxdy = j\omega\int_S \Delta\varepsilon(y)\boldsymbol{E}_1 \cdot \boldsymbol{E}_2 dxdy + \int_S \boldsymbol{E}_2 \cdot \boldsymbol{J}_1 dxdy - \int_S \boldsymbol{E}_1 \cdot \boldsymbol{J}_2 dxdy \tag{6}$$

where $\hat{\boldsymbol{z}}$ is the unit vector in the z direction and S denotes the cross-section area in the transverse $x-y$ plane. Equation (6) constitutes the basis of the coupled-mode formulation presented here.

To illustrate the formulation process, we consider two coupled conductor transmission lines a and b embedded in a trilayered dielectric medium over a perfectly conducting ground plane as shown in Fig. 1. C_a and C_b represent the cross-sectional contours of perfect conductors a and b in the transverse plane. The geometry is uniform in the wave-guiding z direction and the permittivity distribution of the dielectric layer is denoted by $\varepsilon(y)$. We define two sets of solutions of Maxwell's equations to which the reciprocity relation (6) is applied. For the first set of solutions $(\boldsymbol{E}_1, \boldsymbol{H}_1, \boldsymbol{J}_1)$, we adopt the eigenmode fields and currents $(\boldsymbol{E}, \boldsymbol{H}, \boldsymbol{J})$ for the original coupled structure as shown in Fig. 1 with $\varepsilon_1(y) = \varepsilon(y)$. As the second set of solutions $(\boldsymbol{E}_2, \boldsymbol{H}_2, \boldsymbol{J}_2)$, we employ the eigenmode fields and currents associated with the isolated single conductor lines a and b located in the dielectric media with $\varepsilon_2(y) = \varepsilon_a(y)$ and $\varepsilon_2(y) = \varepsilon_b(y)$, respectively. In this case, two different models of the isolated conductor lines are possible as the basis of the original coupled structure. Fig. 2(a) and (b) show the isolated conductor lines located in the same trilayered dielectric medium as in the original structure, for which $\varepsilon_a(y) = \varepsilon_b(y) = \varepsilon(y)$. Fig. 3(a) and (b) show the isolated conductor lines located in the two-layered dielectric media with $\varepsilon_a(y)$ and $\varepsilon_b(y)$, respectively, that are different from the original one with $\varepsilon(y)$. The eigenmode solutions for the fundamental modes in those isolated configurations (a) and (b) are denoted as follows:

$$\begin{aligned}\boldsymbol{E}_\nu^{(\pm)} &= \boldsymbol{e}_\nu^{(\pm)}(x,y)\exp(\mp j\beta_\nu z)\\ &= [\boldsymbol{e}_{\nu,t}(x,y) \pm \hat{\boldsymbol{z}} e_{\nu,z}(x,y)]\exp(\mp j\beta_\nu z)\end{aligned} \quad (7)$$

$$\begin{aligned}\boldsymbol{H}_\nu^{(\pm)} &= \boldsymbol{h}_\nu^{(\pm)}(x,y)\exp(\mp j\beta_\nu z)\\ &= [\pm\boldsymbol{h}_{\nu,t}(x,y) + \hat{\boldsymbol{z}} h_{\nu,z}(x,y)]\exp(\mp j\beta_\nu z)\end{aligned} \quad (8)$$

$$\begin{aligned}\boldsymbol{J}_\nu^{(\pm)} &= \boldsymbol{j}_\nu^{(\pm)}(x,y)\exp(\mp j\beta_\nu z)\\ &= [\boldsymbol{j}_{\nu,t}(x,y) \pm \hat{\boldsymbol{z}} j_{\nu,z}(x,y)]\exp(\mp j\beta_\nu z)\end{aligned} \quad (9)$$

where $\nu = a, b$, the superscripts $(+)$ and $(-)$ indicate the modes propagating in the $+z$ and $-z$ directions, respectively, and β_ν is the propagation constant of the isolated conductor line ν. Applying the reciprocity relation (6) to the two sets of solutions $(\boldsymbol{E}, \boldsymbol{H}, \boldsymbol{J})$ and $(\boldsymbol{E}_a^{(-)}, \boldsymbol{H}_a^{(-)}, \boldsymbol{J}_a^{(-)})$, we obtain

$$\begin{aligned}&\frac{\partial}{\partial z}\int_S (\boldsymbol{E}\times\boldsymbol{H}_a^{(-)} - \boldsymbol{E}_a^{(-)}\times\boldsymbol{H})\cdot\hat{\boldsymbol{z}}\,dxdy\\ &= j\omega\int_S \Delta\varepsilon_a(y)\boldsymbol{E}\cdot\boldsymbol{E}_a^{(-)}dxdy + \int_{C_b}\boldsymbol{E}_a^{(-)}\cdot\boldsymbol{J}dxdy\end{aligned} \quad (10)$$

where $\Delta\varepsilon_a(y) = \varepsilon(y) - \varepsilon_a(y)$, and we have used the boundary conditions that $\boldsymbol{E}\cdot\boldsymbol{J}_a^{(-)} = 0$ on C_a and C_b, and $\boldsymbol{E}_a^{(-)}\cdot\boldsymbol{J} = 0$ on C_a. Similarly two sets of solutions $(\boldsymbol{E}, \boldsymbol{H}, \boldsymbol{J})$ and $(\boldsymbol{E}_b^{(-)}, \boldsymbol{H}_b^{(-)}, \boldsymbol{J}_b^{(-)})$ lead to

$$\begin{aligned}&\frac{\partial}{\partial z}\int_S (\boldsymbol{E}\times\boldsymbol{H}_b^{(-)} - \boldsymbol{E}_b^{(-)}\times\boldsymbol{H})\cdot\hat{\boldsymbol{z}}\,dxdy\\ &= j\omega\int_S \Delta\varepsilon_b(y)\boldsymbol{E}\cdot\boldsymbol{E}_b^{(-)}dxdy + \int_{C_a}\boldsymbol{E}_b^{(-)}\cdot\boldsymbol{J}dxdy\end{aligned} \quad (11)$$

where $\Delta\varepsilon_b(y) = \varepsilon(y) - \varepsilon_b(y)$, and we have used the boundary conditions that $\boldsymbol{E}\cdot\boldsymbol{J}_b^{(-)} = 0$ on C_a and C_b, and $\boldsymbol{E}_b^{(-)}\cdot\boldsymbol{J} = 0$ on C_b. Although (10) and (11) give the exact relations, $(\boldsymbol{E}, \boldsymbol{H}, \boldsymbol{J})$ are the unknown eigenmode fields and current for the coupled system. Then we express those fields and current distribution as follows:

$$\boldsymbol{E} = a(z)\boldsymbol{e}_a^{(+)}(x,y) + b(z)\boldsymbol{e}_b^{(+)}(x,y) \quad (12)$$

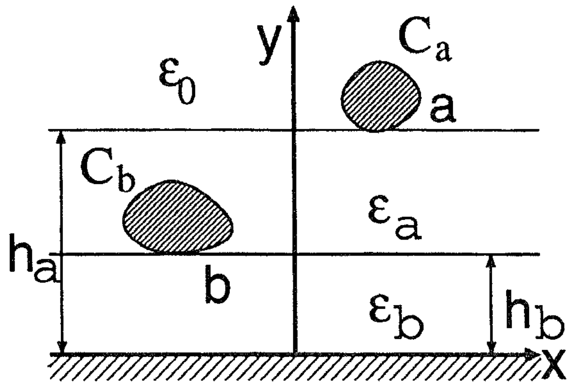

Fig. 1. General configuration of coupled two-conductor lines located in a trilayered dielectric medium with $\varepsilon(y)$.

$$\boldsymbol{H} = a(z)\boldsymbol{h}_a^{(+)}(x,y) + b(z)\boldsymbol{h}_b^{(+)}(x,y) \quad (13)$$

$$\boldsymbol{J} = a(z)\boldsymbol{j}_a^{(+)}(x,y) + b(z)\boldsymbol{j}_b^{(+)}(x,y) \quad (14)$$

where $a(z)$ and $b(z)$ are unknown amplitude functions. The above expressions are just the modal expansions in terms of the two fundamental modes in isolated single conductor lines a and b. It is also noted that the above expansion is only an approximate set of solutions to the eigenmode fields and current of the coupled system and the electric field defined by (12) satisfies, only approximately, the boundary conditions on the coupled conductor surfaces.

Substituting (7)–(9) and (12)–(14) into (10) and (11) and taking into account that $\boldsymbol{j}_a^{(+)} = 0$ on C_b and $\boldsymbol{j}_b^{(+)} = 0$ on C_a, the coupled-mode equations governing the evolution of the amplitude functions $a(z)$ and $b(z)$ are derived. For the first choice of the isolated conductor lines with $\Delta\varepsilon_a(y) = \Delta\varepsilon_b(y) = 0$ as shown in Fig. 2, we obtain

$$\begin{aligned}&\frac{d}{dz}a(z) + \frac{N_{ab}+N_{ba}}{2}\frac{d}{dz}b(z)\\ &= -j\beta_a a(z) - j\left[\beta_a\frac{N_{ab}+N_{ba}}{2} + K_{ab}\right]b(z)\end{aligned} \quad (15)$$

$$\begin{aligned}&\frac{d}{dz}b(z) + \frac{N_{ab}+N_{ba}}{2}\frac{d}{dz}a(z)\\ &= -j\beta_b b(z) - j\left[\beta_b\frac{N_{ab}+N_{ba}}{2} + K_{ba}\right]a(z)\end{aligned} \quad (16)$$

with

$$N_{\nu\mu} = \frac{1}{2}\int_S [\boldsymbol{e}_\nu(x,y)\times\boldsymbol{h}_\mu(x,y)]\cdot\hat{\boldsymbol{z}}\,dxdy \quad (17)$$

$$\begin{aligned}K_{\nu\mu} = -\frac{j}{4}\int_{C_\mu}&[\boldsymbol{e}_{\nu,t}(x,y)\cdot\boldsymbol{j}_{\mu,t}(x,y)\\ &- e_{\nu,z}(x,y)j_{\mu,z}(x,y)]dxdy\end{aligned} \quad (18)$$

where $\nu, \mu = a, b$, and we have assumed that the eigenmode fields are normalized so that $N_{aa} = N_{bb} = 1$. For the second choice of the isolated conductor lines as shown in Fig. 3, we have

$$\begin{aligned}&\frac{d}{dz}a(z) + \frac{N_{ab}+N_{ba}}{2}\frac{d}{dz}b(z)\\ &= -j[\beta_a + L_{aa}]a(z)\\ &\quad - j\left[\beta_a\frac{N_{ab}+N_{ba}}{2} + L_{ab} + K_{ab}\right]b(z)\end{aligned} \quad (19)$$

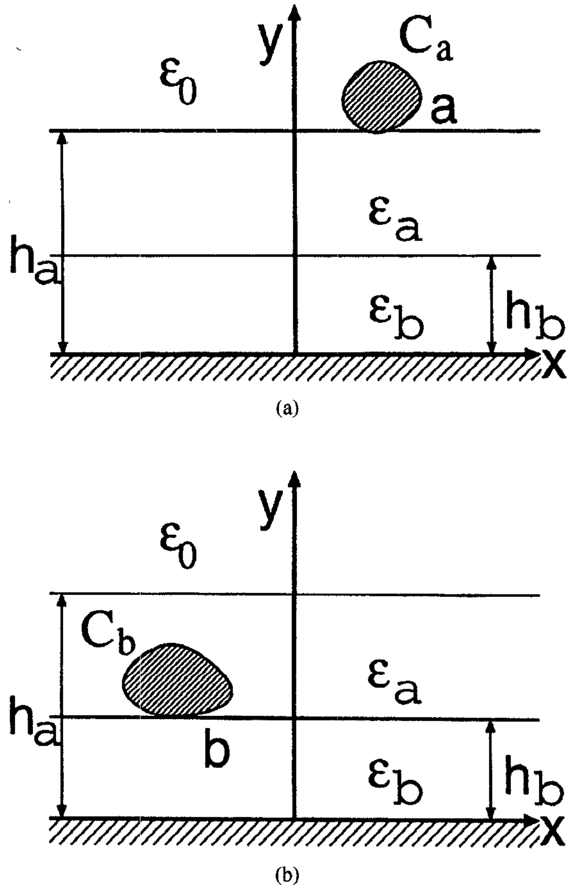

Fig. 2. (a) Isolated conductor line "a" and (b) isolated conductor line "b" located in the same dielectric medium with $\varepsilon(y)$ as the original structure in Fig. 1.

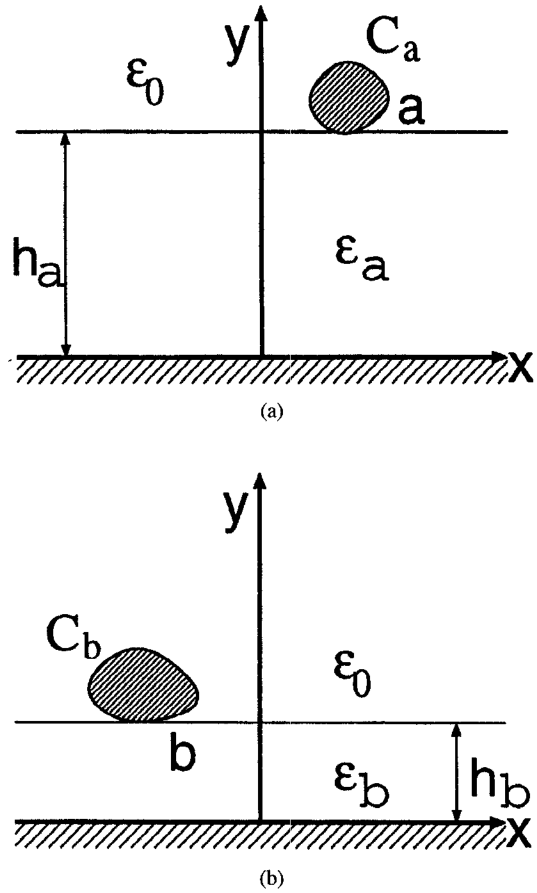

Fig. 3. (a) Isolated conductor line "a" located in a two-layered dielectric medium with $\varepsilon_a(y)$ and (b) isolated conductor line "b" located in another two-layered dielectric medium with $\varepsilon_b(y)$.

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\begin{aligned}
&\frac{d}{dz}b(z) + \frac{N_{ab}+N_{ba}}{2}\frac{d}{dz}a(z) \\
&= -j[\beta_b + L_{bb}]b(z) \\
&\quad - j[\beta_b \frac{N_{ab}+N_{ba}}{2} + L_{ba} + K_{ba}]a(z)
\end{aligned} \tag{20}
$$

with

$$
\begin{aligned}
L_{\nu\mu} = \frac{\omega}{4}\int_S &\Delta\varepsilon_\nu(y)[\boldsymbol{e}_{\nu,t}(x,y)\cdot\boldsymbol{e}_{\mu,t}(x,y) \\
&- e_{\nu,z}(x,y)e_{\mu,z}(x,y)]dxdy
\end{aligned} \tag{21}
$$

where $\nu, \mu = a, b$. Thus, the problem of two coupled conductor lines is reduced to that of two isolated conductor lines. When the eigenmode fields and currents for the isolated conductor lines a and b are specified analytically or numerically for the respective propagation models, the coupling coefficients $N_{\nu\mu}$, $K_{\nu\mu}$, and $L_{\nu\mu}$ $(\nu, \mu = a, b)$ governing the interaction between the two conductor lines are easily calculated by the overlap integrals given by (17), (18), and (21). The solutions to the coupled-mode equations (15) and (16) or (19) and (20) give the propagation constants of two fundamental modes in the coupled conductor lines. Comparing the coupled-mode formulation based on the two different propagation models, we note that the calculation of eigenmodes of isolated conductor lines as the basis, is simpler in the second propagation model, whereas the resulting coupled-mode equations are more complicated than those obtained by the first propagation model.

The procedure of coupled-mode formulation presented here is general. For a system of multilayered N coupled-conductor transmission lines, we may introduce N configurations of isolated single conductor line placed in an appropriate dielectric medium as the basis of propagation model. The eigenmode fields and current of the coupled system are approximated by a linear combination of those of isolated N systems. The interaction between the νth and μth conductor lines are described by the coupling coefficients $N_{\nu\mu}$, $K_{\nu\mu}$, and $L_{\nu\mu}$ defined by (17), (18), and (21). Then the problem of the N coupled-conductor lines is reduced to the analysis of the N isolated single conductor lines and the simple numerical integrations for calculating the coupling coefficients. This analytical and numerical procedure is much simpler than the direct numerical solution methods. It is noted that although various numerical techniques have been developed [1] for the analysis of coupled-conductor lines, the application of those methods to the configurations of multiple nonidentical coupled lines are very complicated and rather difficult.

III. Numerical Examples

As a basic example, the proposed coupled-mode theory was used to analyze coupled-microstrip lines as shown in Fig. 4. Two identical microstrips a and b of width $2w$ and zero thickness are situated with spacing $2d$ on the substrate-cover

TABLE I
NORMALIZED PROPAGATION CONSTANTS β/k_0 OF THE SYMMETRIC AND ASYMMETRIC EH_0 MODES OF TWO COUPLED MICROSTRIP LINES WITH $w = 1.5$ mm, $h = 0.635$ mm, $\varepsilon_r = 9.8$, $f = 5 \sim 20$ GHz, AND VARIOUS SEPARATION DISTANCES d/w. β_0 IS THE PROPAGATION CONSTANT OF THE EH_0 MODE OF ISOLATED SINGLE MICROSTRIP AND k_0 IS THE WAVENUMBER IN FREE SPACE. CMT AND MOM REFER TO THE PRESENT COUPLED-MODE THEORY AND THE DIRECT GALERKIN'S MOMENT METHOD

(a) $f = 5GHz$ $(\beta_0/k_0 = 2.83466)$

	Symmetric mode				Asymmetric mode			
d/w	1.10	1.30	1.50	2.00	1.10	1.30	1.50	2.00
CMT	2.96462	2.93887	2.91099	2.87032	2.68284	2.72413	2.75571	2.79826
MOM	2.95881	2.93332	2.90836	2.86994	2.68033	2.72062	2.75351	2.79775

(b) $f = 10GHz$ $(\beta_0/k_0 = 2.89439)$

	Symmetric mode				Asymmetric mode			
d/w	1.10	1.30	1.50	2.00	1.10	1.30	1.50	2.00
CMT	3.01131	2.98093	2.95173	2.91388	2.75360	2.79981	2.83363	2.87432
MOM	3.00504	2.97713	2.95055	2.91397	2.75194	2.79690	2.83183	2.87393

(c) $f = 15GHz$ $(\beta_0/k_0 = 2.94191)$

	Symmetric mode				Asymmetric mode			
d/w	1.10	1.30	1.50	2.00	1.10	1.30	1.50	2.00
CMT	3.03885	3.00733	2.98062	2.95115	2.82315	2.86947	2.90071	2.93247
MOM	3.03443	3.00622	2.98092	2.95147	2.82488	2.86838	2.89971	2.93225

(d) $f = 20GHz$ $(\beta_0/k_0 = 2.97776)$

	Symmetric mode				Asymmetric mode			
d/w	1.10	1.30	1.50	2.00	1.10	1.30	1.50	2.00
CMT	3.05624	3.02606	3.00332	2.98207	2.88120	2.92425	2.95070	2.97340
MOM	3.05437	3.02685	3.00422	2.98227	2.88632	2.92450	2.95011	2.97320

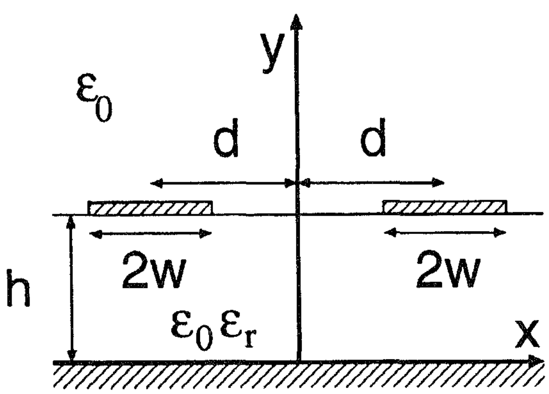

Fig. 4. Configuration of two identical coupled microstrip lines.

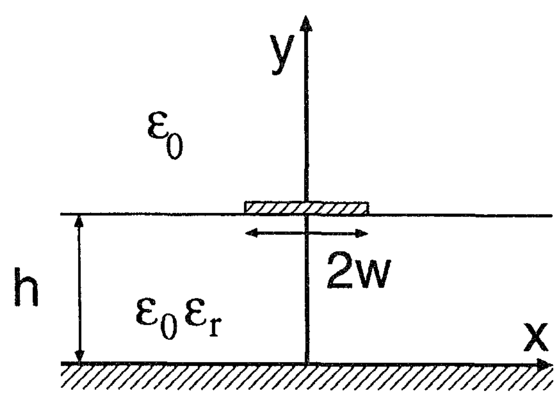

Fig. 5. Conventional single microstrip line introduced as the basis of coupled microstrip lines shown in Fig. 4.

interface in a trilayered structure, which consists of a ground plane of perfect conductor, a dielectric substrate of thickness h and relative permittivity ε_r, and a cover layer of free space. For this trilayered structure, two configurations for the isolated conductor line model shown in Figs. 2 and 3 coincide with each other, and the coupled-mode equations (19) and (20) are reduced to (15) and (16) with $\Delta\varepsilon_a(y) = \Delta\varepsilon_b(y) = 0$.

The eigenmode fields and currents of the isolated single microstrips a and b, which are used to evaluate the coupling coefficients $K_{\nu\mu}$ and $N_{\nu\mu}$ $(\nu, \mu = a, b)$, can be easily calculated with Galerkin's moment method solutions [3] in the spectral domain for a conventional single microstrip as shown in Fig. 5. Let β_0, $\tilde{e}_0(\zeta, y)$ and $\tilde{h}_0(\zeta, y)$, and $\tilde{j}_0(\zeta)$ be the propagation constant, the eigenmode fields, and the eigenmode current in the Fourier transformed domain for the fundamental EH_0 mode of the conventional single microstrip line. Then we have the following relations

$$\beta_a = \beta_b = \beta_0 \tag{22}$$

$$[\tilde{e}_a(\zeta, y), \tilde{h}_a(\zeta, y), \tilde{j}_a(\zeta)] = [\tilde{e}_0(\zeta, y), \tilde{h}_0(\zeta, y), \tilde{j}_0(\zeta)] \exp(j\zeta d) \tag{23}$$

$$[\tilde{e}_b(\zeta, y), \tilde{h}_b(\zeta, y), \tilde{j}_b(\zeta)] = [\tilde{e}_0(\zeta, y), \tilde{h}_0(\zeta, y), \tilde{j}_0(\zeta)] \exp(-j\zeta d). \tag{24}$$

These solutions are substituted into (17) and (18). Omitting the mathematical details, the coupling coefficients are given

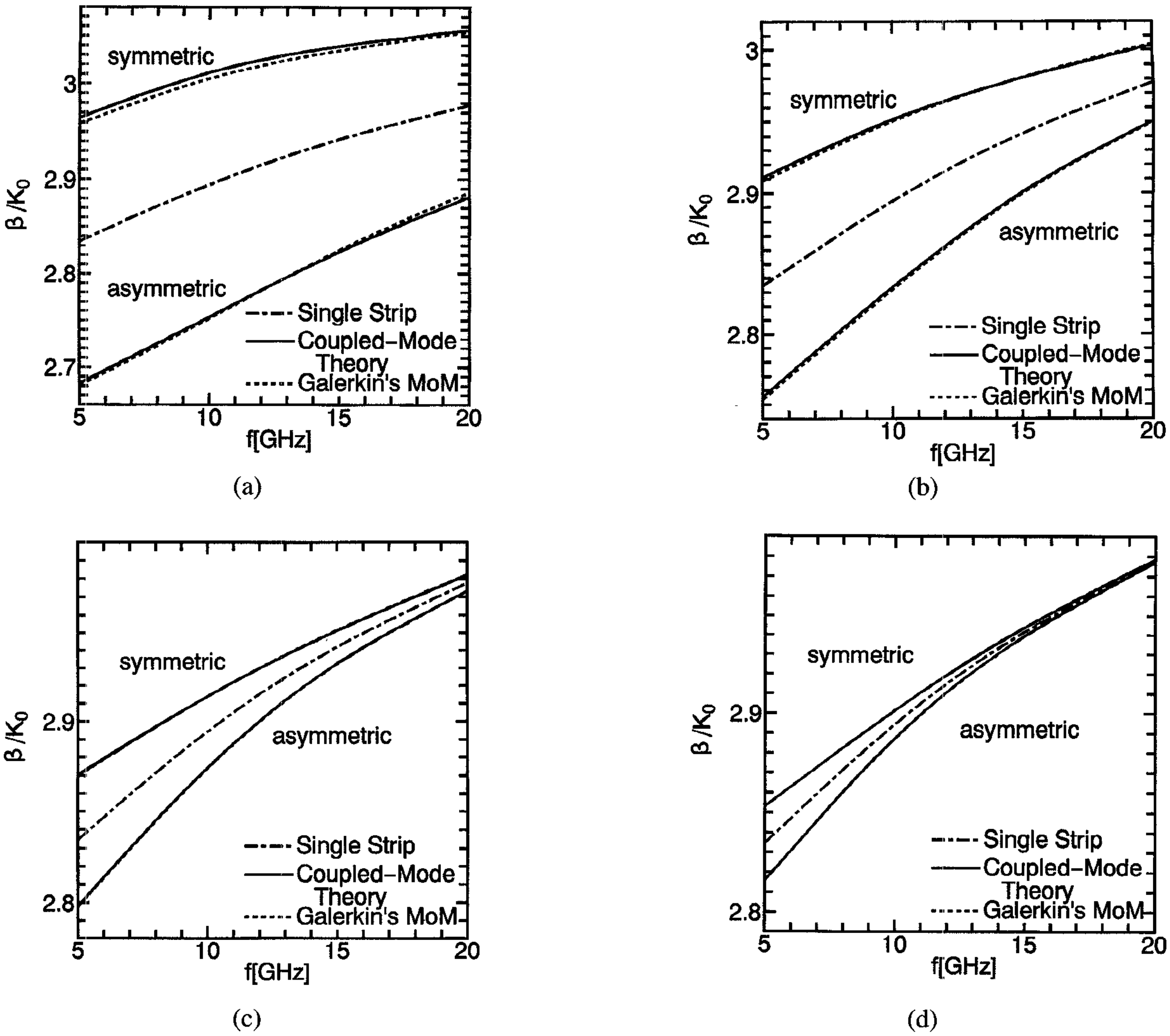

Fig. 6. Dispersion characteristics of the symmetric and asymmetric EH_0 modes of two coupled microstrip lines for four different separations. The values of parameters are the same as those given in Table I. (a) $d/w = 1.1$. (b) $d/w = 1.5$. (c) $d/w = 2.0$. (d) $d/w = 2.5$.

as follows:

$$N = \frac{N_{ab} + N_{ba}}{2} = \int_0^\infty I(\zeta)\cos(2\zeta d)d\zeta \tag{25}$$

$$\begin{aligned} K &= K_{ab} = K_{ba} \\ &= \frac{j}{4\pi}\int_0^\infty [\tilde{e}_{0,x}(\zeta,h)\tilde{j}_{0,x}(\zeta) + \tilde{e}_{0,z}(\zeta,h)\tilde{j}_{0,z}(\zeta)] \\ &\quad \times \cos(2\zeta d)d\zeta \end{aligned} \tag{26}$$

with

$$I(\zeta) = -\frac{2}{\pi}\int_0^\infty [\tilde{e}_{0,x}(\zeta,y)\tilde{h}_{0,y}(\zeta,y) + \tilde{e}_{0,y}(\zeta,y)\tilde{h}_{0,x}(\zeta,y)]dy \tag{27}$$

where we have used the symmetric properties of $\tilde{e}_0(\zeta, y)$, $\tilde{\boldsymbol{h}}_0(\zeta, y)$, and $\tilde{\boldsymbol{j}}_0(\zeta)$ as the functions of ζ. The integral in (27) can be evaluated in closed form using the dyadic Green's function [5] in the spectral domain. The integrals in (25) and (26) are efficiently calculated using the spectral data which were obtained in Galerkin's moment method analysis of the conventional single microstrip line as shown in Fig. 5. Note that the coupling effect between two microstrip lines is described by the integrand factor $\cos(2\zeta d)$ in (25) and (26). The coupled-mode equations (15) and (16) with the substitution of (22), (25), and (26) give two solutions. One is the symmetric EH_0 mode with the propagation constant

$$\beta = \beta_0 + \frac{K}{1+N} \tag{28}$$

which yields $a(z) = b(z)$, and the other is the asymmetric EH_0 mode with the propagation constant

$$\beta = \beta_0 - \frac{K}{1-N} \tag{29}$$

which yields $a(z) = -b(z)$.

The method described above was implemented to calculate the propagation constants of two identical coupled microstrip lines. For comparison, the same coupled problem was also rigorously solved by using directly Galerkin's moment method [3] with Chebyshev polynomial basis functions weighted by appropriate edge factors. The normalized propagation constants β/k_0 of the symmetric and asymmetric EH_0 modes calculated by (28) and (29) are given in Table I for $w = 1.5$ mm, $h = 0.635$ mm, $\varepsilon_r = 9.8$, $f = 5 \sim 20$ GHz, and various separations d/w, and compared with those of the direct Galerkin's moment method solutions, where k_0 is the wavenumber in free space. Fig. 6(a)–(d) shows the dispersion curves of the symmetric and asymmetric EH_0 modes of coupled microstrips with four different separations. The solid lines are the results of the present coupled-mode theory and the dashed lines are those of the direct Galerkin's moment method. The values of parameters are the same as those given in Table I. Comparing the results in Table I and Fig. 6 obtained by the two different approaches, we can see that the coupled-mode approximations are in very close agreement with the rigorous Galerkin's moment method solutions over a broad range of weak to strong coupling. Due to the simpler matrix equation involved, the numerical procedure of the coupled-mode analysis is much more efficient than that of the direct Galerkin's moment method. For the same computation of propagation constants, the coupled-mode analysis requires only about 8% of the computer time needed by the direct moment method.

The rigorous Galerkin's moment method solutions show that when the two microstrips are widely separated, the propagation constants of the coupled system converge to that of the corresponding isolated single microstrip. As the two microstrips become closer, the coupled system modes, symmetric and asymmetric modes, emerge and their propagation constants shift nearly symmetrically from the isolated one. When the microstrip separation is further decreased, the shifts become asymmetrical. The coupled-mode solutions (28) and (29) clearly describe this situation. The coupling coefficient N explains such an asymmetric behavior of the propagation constants of the coupled modes in a strongly coupled case. In this respect, the present coupled-mode theory is different from the full-wave perturbation theory[3]. Although the perturbation theory is another efficient approximate technique, it predicts the coupled system modes shifted symmetrically from the isolated one over the entire separation distance and loses the validity in the strongly coupling regime.

Before concluding, it is worth mentioning why the present coupled-mode formulation yields a very good approximation for the propagation constants of coupled microstrip lines. As stated in Section II, the assumed expansions of the eigenmode fields and current (12) to (14) do not satisfy fully Maxwell's equations and the boundary conditions for the coupled-conductor lines. However, the propagation constants has a stationary nature with respect to a small variation in the associated field distributions. Following the same procedure as in the coupled optical waveguides [4], we can derive the variational expression for the propagation constants which leads to the identical coupled-mode equations as (15) and (16) or (19) and (20). This fact indicates that any deviations of first order in the assumed eigenmode field distributions only result in errors of second order in the propagation constants.

IV. Conclusion

A novel coupled-mode theory for multilayered and multiconductor transmission lines has been developed based on the generalized reciprocity relation. This coupled-mode theory is a powerful analytical and numerical technique for approximating the coupling between adjacent conductor lines with a good physical justification. The coupling coefficients are given by the simple overlap integrals between the eigenmode fields and currents associated with individual isolated single lines. This greatly simplifies the computational procedure and therefore remarkably reduces computation time. The proposed coupled-mode theory was applied to the analysis of two identical coupled-microstrip lines. The numerical results of the propagation constants are in very close agreement with those of the rigorous Galerkin's moment method solutions over a broad range of weak to strong coupling, indicating that the coupled-mode theory yields a good approximation with enough accuracy. One disadvantage of the present coupled-mode theory is that it does not give the correct coupled-current distributions. This is because the assumed electric field defined by (12) does not satisfy fully the boundary conditions on the coupled conductor surfaces. The coupled-current distributions can be calculated when the more rigorous perturbation technique is implemented for the coupled-mode formulation. One such method is the coupled-mode theory based on a singular perturbation scheme [6].

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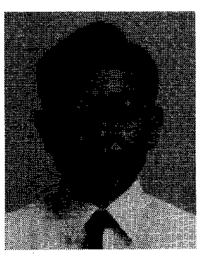

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